Welfare Implications of a Discretized Sequential Hotelling Firm Location Choice Model

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Research Question

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What are the consequences of introducing sequential firm entry into Hotelling's location choice model?

Goals

1) Computationally solve the firms' location decisions in a discrete $[0, 1]$ space.
2) Examine how consumer welfare can be affected by the firms' sequential location choices.
3) Consider the effects of government interventions on consumer welfare.

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Model

- Sequential version of Hotelling's location choice model.
- Discrete $[0, 1]$ space.
- Finite number of firms.
  - Two Stages:
    - (First Stage) Firms enter sequentially. Each firm chooses from the finite set of available locations to maximize expected profits.
    - (Second Stage) Firms take locations as given and simultaneously set prices.
  - Firm j's Profit Maximization Problem:

$$\max_{p_j} \pi_j = (p_j - c)q_j - f$$

    - Fixed (sunk) entry cost f
    - Marginal cost c
    - q_j is firm j's residual demand

Model

- Homogenous consumers
  - Uniformly distributed along the measure one space $[0, 1]$
  - Each consumer i at location x_i receives the following surplus net of travel cost when purchasing a product from firm j:

$$U_i = V - p_j - t(x_i - i_j)^2$$

    - V is the valuation of the product
    - p_j is the price of the product sold by firm j
    - i_j is the location of firm j
  - Unit elastic demand
  - V is sufficiently large such that they will always buy from some firm.

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Algorithm I

1. Set exogenous variables N_f (number of firms), N_l (number of locations), c (variable cost), t (travel cost), and f (fixed cost). In the case with a fixed number of firms, set $f = 0$ without loss of generality.

2. For *each possible set* of firm locations, solve *each firm's* analytical profit function.[1]
   - 2.1 Given firm locations, analytically solve each consumer's preferred firm (as a function of prices). In particular, this is accomplished by solving the locations of the $N_f - 1$ indifferent consumers.
   - 2.2 Firms choose price to maximize profits, accounting for other firms' profit maximizing prices and the locations of indifferent consumers. In particular, take first order conditions of each firm's profit function to get a system of N_f equations.
   - 2.3 Analytically solve the system of N_f equations to solve optimal p_i^*, and consequently optimal π_i^*, for each firm $i \in \{1, \ldots, N_f\}$.

3. Using backward induction, solve each firm's best location response function:
   - 3.1 Start with firm N_f: set $i = N_f$.
   - 3.2 Consider all firm location combinations for firms $j < i$.
   - 3.3 For each combination, place firm i in each open location.
   - 3.4 For each location, best response functions for firms $j > i$ are known.

Algorithm II

3.5 Using each location combination for firms $j < i$; each location choice for firm i; and best response functions for firms $j > i$; calculate profits for firm i using the results from b. It is possible firms $j > i$ have multiple best responses: if this occurs, calculate expected profits taking all indifferent location decisions as equally likely.

3.6 For each location combination for firms $j < i$, take the location choice (choices) that maximizes (maximize) profits for firm i, setting this (these) as its best response (responses). If the maximum profit is negative, we assume firm i chooses not to enter the market (this cannot occur for the fixed number of firms case, as $f = 0$).

3.7 Once firm i's best response is known for each location combination for firms $j < i$, if $i > 1$ then decrease i by 1 and go back to step (b). If $i = 1$, then the process is complete.

4. Take firm 1's best response function and determine which location (or locations) maximize its expected profits. Iterate through each possible combination of best responses for firms $i > 1$ to get all possible equilibrium outcomes.

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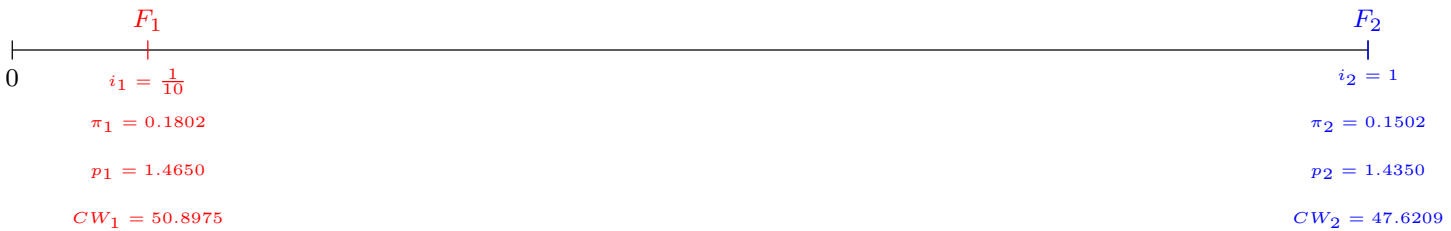
Results 1: Increasing Fixed Cost for Endogenous Firm Entry I

Figure: $f = \frac{4}{50}$ (11 Locations), $CW = 98.4584$, $PS = 0.3400$, $TS = 98.7984$

Figure: $f = \frac{3}{50}$ (11 Locations), $CW = 98.5184$, $PS = 0.3304$, $TS = 98.8504$



Results 1: Increasing Fixed Cost for Endogenous Firm Entry II

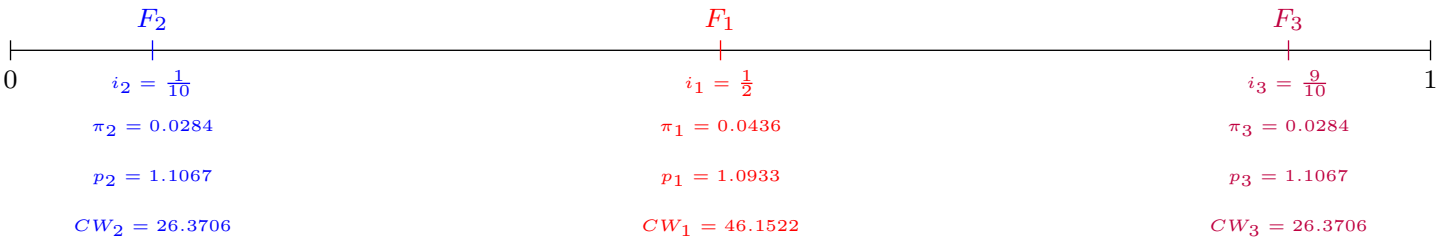
Figure: $f = \frac{2}{50}$ (11 Locations), $CW = 98.6262$, $PS = 0.2735$, $TS = 98.8935$

Figure: $f = \frac{1}{50}$ (11 Locations), $CW = 98.7836$, $PS = 0.1609$, $TS = 98.9409$

Results 2: Increasing Number of Fixed Firm Entry I

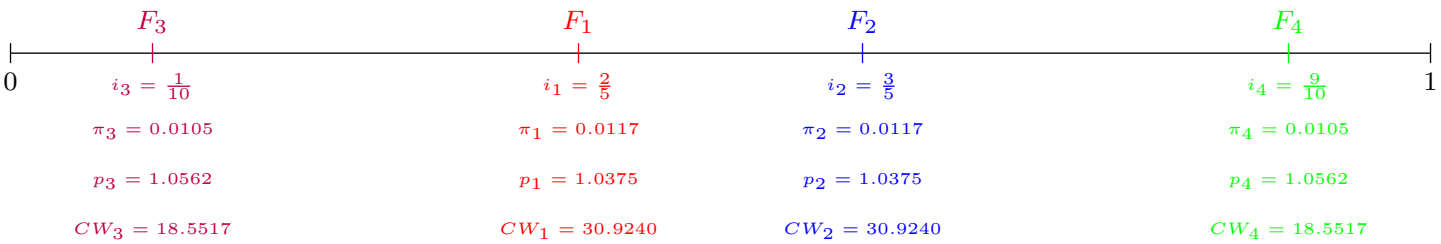
Figure: 2 Firms (11 Locations), $CW = 98.4584$, $PS = 0.5000$, $TS = 98.9584$

Figure: 3 Firms (11 Locations), $CW = 98.8934$, $PS = 0.1004$, $TS = 98.9938$



Results 2: Increasing Number of Fixed Firm Entry II

Figure: 4 Firms (11 Locations), $CW = 98.9514$, $PS = 0.0444$, $TS = 98.9958$



Results 3: Increasing Number of Available Locations I

Figure: 3 Firms (11 Locations), $CW = 98.8934$, $PS = 0.1004$, $TS = 98.9904$

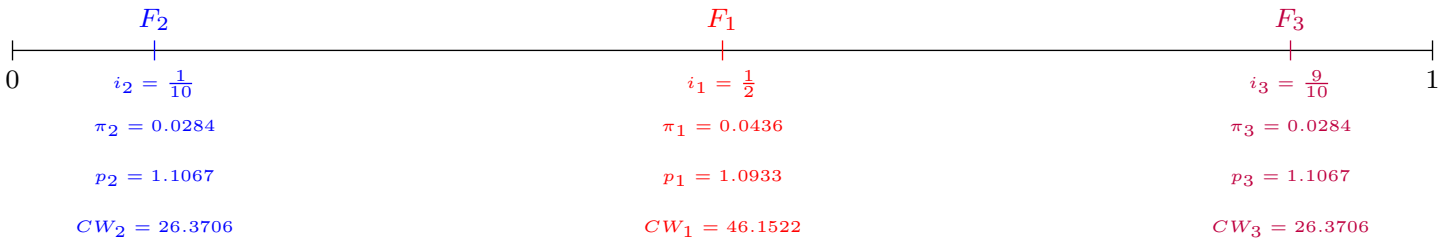
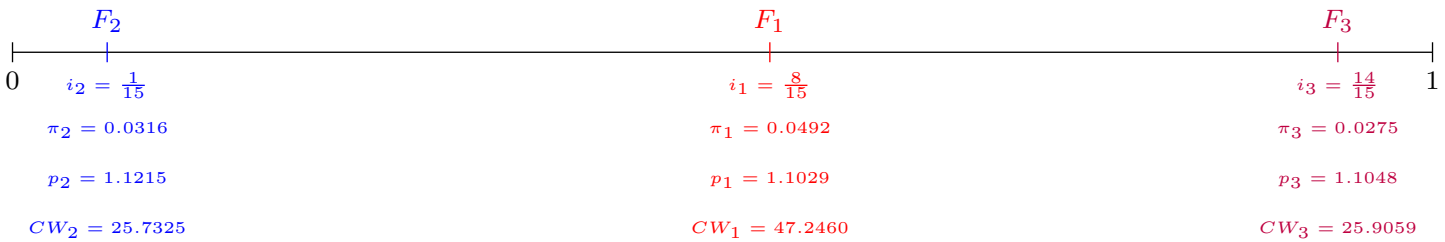
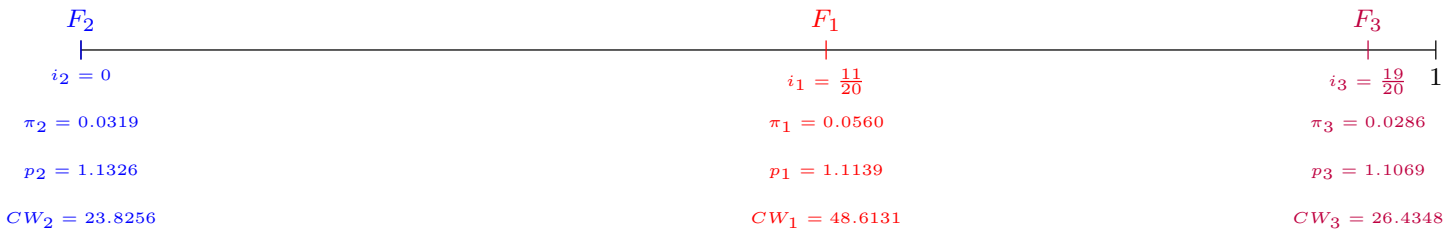


Figure: 3 Firms (16 Locations), $CW = 98.8844$, $PS = 0.1083$, $TS = 98.9883$



Results 3: Increasing Number of Available Locations II

Figure: 3 Firms (21 Locations), $CW = 98.8735$, $PS = 0.1165$, $TS = 98.9865$



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Discussion and Conclusion

1) Increasing fixed cost for endogenous firm entry:
   - Decrease in fixed cost $\rightarrow$ firms become more equidistant across $[0, 1]$ space and decrease price $\rightarrow$ increase consumer welfare.
   - Shift represent strategic decision to deter future firm entry.
2) Increasing number of fixed firm entry:
   - Similar results to simultaneous setting - relatively equidistantly located firms.
   - Early entrants have higher profits than late entrants.
3) Increasing number of available locations:
   - Give early entrants more location choice $\rightarrow$ increase in profits.
   - Decrease in consumer welfare.
   - Decrease in consumer welfare $>$ increase in profits $\rightarrow$ net decrease in total welfare.

Government Implications ?