Macroeconomic Impact of Population Aging in Japan Using an Overlapping Generations Model *

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Abstract

This paper simulates the effect of the demographic transition in Japan on aggregate macroeconomic outputs. Our model uses the open source overlapping generations general equilibrium model provided by DeBacker and Evans (2018). We calibrate the model to include Japanese demographics, tax rates, and labor parameters, including the elasticity and disutility of labor.

Our simulations indicate Japan's declining fertility rates result in a modal population age of around 70 by 2100. An aging population and declining workforce decrease government revenue while increasing government spending, making current government spending patterns unsustainable in the long-term. Our steady-state results require negative government spending. This implies tax revenues will not be able to cover anticipated government spending in the future.

keywords: Overlapping Generations Model, Demographic Transitions, Japanese Calibration

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1 Introduction

Japan's declining fertility rate and stable mortality rate imply drastic changes in demographics over the coming decades. Our simulation expects the modal age to be around 70 years by 2100.[1] These demographic effects have real implications on the macro-economy. The increased payments for retirement benefits combined with the decreased tax revenue and social security contributions from the working-age population have exacerbated Japan's public deficit situation.

Analyzing the effects of the Japanese demographic changes in an overlapping generations setting has not yet been thoroughly investigated. The literature is limited to Muto et al. (2016), who carry out overlapping generations analysis of the effects of these demographic shifts on long-run aggregate outcomes for Japan. They find that decreasing fertility rates contribute to decreasing aggregate output, while increasing longevity counteracts this by increasing aggregate output. However, this increase is not enough to create a net positive effect. Overall, their simulation predicts negative effects on aggregate outputs during the next few decades.

Our model adds to this literature by (1) introducing steady-state population dynamics rather than only looking at population estimates and (2) adding savings in the utility function. In (2), above, this is equivalent to a bequest motive, which Muto et al. (2016) say will better model savings motives towards the end of life. This is because a bequest motive encourages both young and old to save more, which is represented in the data.

Our model calibrates OG-USA, the open source overlapping generations model created by DeBacker and Evans (2018), to Japan. Our calibration provides insight into the sustainability of Japanese government spending given the current demographic transition. In the long-run steady state, Japanese government spending becomes negative, indicating that the debt level is too high for tax revenue to be able to cover government spending.

Section 2 outlines our model based on DeBacker and Evans (2018). Section 3 discusses our estimation strategy to calibrate the model to Japan. Section 4 analyzes the steady state aggregate outcome results. Section 5 provides our concluding remarks and extensions.

[1] This can be seen most dramatically in Figure 4.

2 Model

We present our overlapping generations model based on DeBacker and Evans (2018). We consider three agents: household, firm, and government.

2.1 Household

This section follows Chapter 5 of DeBacker and Evans (2018). Households choose lifetime consumption $\{c_{j,s,t+s-1}\}_{s=1}^{S}$, labor supply $\{n_{j,s,t+s-1}\}_{s=1}^{S}$, and savings $\{b_{j,s,t+s-1}\}_{s=1}^{S}$ to maximize the following lifetime utility:

$$\max_{\{(c_{j,s,t}),(n_{j,s,t}),(b_{j,s,t})\}_{s=E+1}^{E+S}} \sum_{s=1}^{S} \beta^{s-1}\left[\prod_{u=E+1}^{E+s}(1-\rho_s)\right] u(c_{j,s,t+s-1}, n_{j,s,t+s-1}, b_{j,s+1,t+s})$$

$$\text{where } u(c_{j,s,t}, n_{j,s,t}, b_{j,s+1,t+1}) \equiv \frac{(c_{j,s,t})^{1-\sigma}-1}{1-\sigma} + e^{g_y t(1-\sigma)}\chi_s^n\left(b\left[1-\left(\frac{n_{j,s,t}}{\tilde{l}}\right)^{v}\right]^{\frac{1}{v}}\right) \tag{1}$$

$$+\chi_j^b \rho_s \frac{(b_{j,s+1,t+1})^{1-\sigma}-1}{1-\sigma} \quad \forall j,t \quad and \quad E+1 \leq s \leq E+S$$

subject to the following budget constraints and non-negativity constraints:

$$c_{j,s,t} + b_{j,s+1,t+1} = (1+r_t)b_{j,s,t} + w_t e_{j,s} n_{j,s,t} + \zeta_{j,s}\frac{BQ_t}{\lambda_j \omega_{s,t}} + \eta_{j,s,t}\frac{TR_t}{\lambda_j \omega_{s,t}} - T_{s,t}$$
$$\forall j,t \quad and \quad s \geq E+1 \quad where \quad b_{j,E+1,t} = 0 \quad \forall j,t \tag{2}$$

The utility function incorporates CRRA utility of consumption, elliptical disutility of labor, and CRRA utility of savings. Of the choice variables, $c_{j,s,t}$ is consumption, $n_{j,s,t}$ is labor supply, and $b_{j,s+1,t+1}$ is savings in period t that will appear on the right side of the budget constraint in period $t+1$. Of the state variables, r_t is the interest rate, w_t is the wage, and $b_{j,s,t}$ is current wealth. j indicates the ability level of the household, which is randomly assigned at birth from a distribution of J ability levels. In our model, $J = 7$. λ_j is the percent of each age group with ability level j and $e_{j,s}$ is a labor-ability factor that depends on ability level and age. We use US labor-ability factor constructed by DeBacker and Evans (2018).

In the budget constraint, BQ_t indicates the total sum of bequests for period t. $\zeta_{j,s}$ gives the fraction of the bequests that are given to ability type j of age s, and $\lambda_j \omega_{s,t}$ gives the total number of households of ability type j and age s in time t. Therefore, this term gives the total quantity of bequests received by a household of ability type j and age s in time t.

The next term similarly gives the total quantity of transfers received by a household of ability type j and age s in time t. TR_t indicates the total sum of transfers for period t. $\eta_{j,s,t}$ gives the fraction of the transfers that are given to households of ability type j and age s in time t, and $\lambda_j \omega_{s,t}$ gives the total number of households of ability type j and age s in time t. The final term, $T_{s,t}$, gives the total tax liability of a household of age s in time t.

2.2 Firm

This section follows Chapter 7 of DeBacker and Evans (2018). Firms have a constant elasticity of substitution production function based on capital K_t and labor L_t.

$$Y_t = F(K_t, L_t) \equiv Z_t \left[(\gamma)^{\frac{1}{\epsilon}} (K_t)^{\frac{\epsilon-1}{\epsilon}} + (1-\gamma)^{\frac{1}{\epsilon}} (e^{g_y t} L_t)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \quad \forall t \tag{3}$$

where Z_t is total factor productivity at time t, γ is the capital income share, ϵ is the constant elasticity of substitution between capital and labor, and g_y is the rate of progress of labor-augmenting technology. Given this production function, the profit function of a representative firm is the following:

$$PR_t = (1 - \tau^{corp}) \left[F(K_t, L_t) - w_t L_t \right] - (r_t + \delta) K_t + \tau^{corp} \delta^\tau K_t \quad \forall t \tag{4}$$

Normalizing prices to unity allows revenue to be given by the production function directly. Labor costs are given by $w_t L_t$, capital costs are given by $(r_t + \delta) K_t$, and depreciation rates are given by δ. We have a flat tax on corporate income τ^{corp}, which only distorts firms' capital demand decision. We also have a refund on depreciation costs, δ^τ.

Taking derivatives with respect to labor and capital, we can solve for optimal wage and interest rates:

$$w_t = e^{g_y t}(Z_t)^{\frac{\epsilon-1}{\epsilon}}\left[(1-\gamma)\frac{Y_t}{e^{g_y t}L_t}\right]^{\frac{1}{\epsilon}} \quad \forall t \tag{5}$$

$$r_t = (1-\tau^{corp})(Z_t)^{\frac{\epsilon-1}{\epsilon}}\left[\gamma\frac{Y_t}{K_t}\right]^{\frac{1}{\epsilon}} - \delta + \tau^{corp}\delta^{\tau} \quad \forall t \tag{6}$$

2.3 Government

This section follows Chapter 3 of DeBacker and Evans (2018). The government in this model levies taxes, provides transfers, and spends on public goods. There is also a closure rule to ensure government debt does not tend toward positive or negative infinity in the long-run.

2.3.1 Government Tax Revenue

Taxes are levied on firms and households. In each case, total tax liability can be considered as an effective tax rate times total income, with the addition of a tax on depreciation expensing for firms. We define government tax revenue to be the sum of the tax revenue from these sources.

$$\begin{aligned} Rev_t = &\underbrace{\tau^{corp}[Y_t - w_t L_t] - \tau^{corp}\delta^{\tau}K_t}_{\text{corporate tax revenue}} \\ &+ \underbrace{\sum_{s=E+1}^{E+S}\sum_{j=1}^{J}\lambda_j\omega_{s,t}\tau^{etr}_{s,t}(x_{j,s,t},y_{j,s,t})(x_{j,s,t}+y_{j,s,t})}_{\text{household tax revenue}} \quad \forall t \end{aligned} \tag{7}$$

2.3.2 Government Budget Constraint

Denoting government debt in period t as D_t, the government budget constraint requires government spending (discretionary, transfers, and interest on debt) equal government income plus borrowing (budget deficit):

$$G_t + TR_t + r_t D_t = Rev_t + (D_{t+1} - D_t) \tag{8}$$

We treat transfers each period as a fixed fraction, α_{tr}, of GDP. We also include a time-dependent multiplier, $g_{tr,t}$ that will equal unity until some period in the future in which it may adjust in order to provide a budget closure rule.

$$TR_t = g_{tr,t}\alpha_{tr}Y_t \quad \forall t \tag{9}$$

We also treat government discretionary spending as a fixed fraction, α_g, of GDP. We include a similar time-dependent multiplier, $g_{g,t}$.

$$G_t = g_{g,t}\alpha_g Y_t \quad \forall t \tag{10}$$

Exact values for α_{tr} and α_g can be see in Table 2.

2.3.3 Government Budget Closure Rule

In order to ensure the model is stationary, government debt cannot tend towards positive or negative infinity in the long-run. This is enforced through a government closure rule. In our model, we adopt a dual approach: we adjust both transfer and discretionary spending in order to stabilize government debt to a fixed fraction of GDP, α_D. In order to simplify the model, we assume that both are adjusted by the same percentage. As both time-dependent multipliers will be equal, we define

$$g_{trg,t} = g_{g,t} = g_{tr,t} \quad \forall t \tag{11}$$

$$\text{where} \quad g_{trg,t} = \begin{cases} 1 & \text{if} \quad t < T_{G1} \\ \frac{[\rho_d \alpha_D Y_t + (1-\rho_d) D_t] - (1+r_t) D_t + Rev_t}{(\alpha_g + \alpha_{tr}) Y_t} & \text{if} \quad T_{G1} \leq t < T_{G2} \\ \frac{\alpha_D Y_t - (1+r_t) D_t + Rev_t}{(\alpha_g + \alpha_{tr}) Y_t} & \text{if} \quad t \geq T_{G2} \end{cases} \tag{12}$$

Before period T_{G1}, the government will allocate the exogenously determined fixed fraction of GDP towards transfer and discretionary spending. Starting in period T_{G1} until the period before T_{G2}, the government will adjust spending to be a convex combination of current spending and the target fraction of GDP, α_D, based on the factor ρ_d. In period T_{G2}, government spending is adjusted to be exactly the fraction α_D of GDP.

2.4 Market Clearance

There are three markets that need to be cleared: the labor market, the capital market, and the good market. Walras' law tells us we need to show clearance for only two markets for all markets to clear. The three market-clearing conditions are the following:

Labor Market:

$$L_t = \sum_{s=E+1}^{E+S} \sum_{j=1}^{J} w_{s,t} \lambda_j e_{j,s} n_{j,s,t} \quad \forall t \tag{13}$$

Capital Market:

$$K_t + D_t = \sum_{s=E+2}^{E+S+1} \sum_{j=1}^{J} (w_{s-1,t-1} \lambda_j b_{j,s,t} + i_s w_{s,t-1} \lambda_j b_{j,s,t}) \quad \forall t \tag{14}$$

Goods Market:

$$Y_t = C_t + K_{t+1} - \left(\sum_{s=E+2}^{E+S+1} \sum_{j=1}^{J} i_s w_{s,t} \lambda_j b_{j,s,t+1} \right) - (1-\delta) K_t + G_t \quad \forall t \tag{15}$$

2.5 Stationary Steady-State and Non Steady-State Equilibrium

We apply the solution method provided by DeBacker and Evans (2018). Please refer to the paper for a detailed description.

3 Calibration/Estimation

3.1 Demographic Dynamics

This section follows Chapter 3 of DeBacker and Evans (2018).

3.1.1 Population Dynamics

Population evolves according to the following:

$$\omega_{1,t+1} = (1-\rho_0)\sum_{s=1}^{E+S} f_s\omega_{s,t} + i_1\omega_{1,t} \quad \forall t \tag{16}$$

$$\omega_{s+1,t+1} = (1-\rho_s)\omega_{s,t} + i_{s+1}\omega_{s+1,t} \quad \forall t \quad \text{and} \quad 1 \leq s \leq E+S-1 \tag{17}$$

where $\omega_{s,t}$ represents population of age s at time t; $f_s \geq 0$, $\rho_s \geq 0$, and i_s represent fertility rates, mortality rates, and immigration rates respectively, at age s; and $E+S$ represent periods of life, where the first E periods are spent not working, then periods $E+1$ to $E+S$ are spent working (assuming the agent does not die). The following sections explain our calibration for these values.

3.1.2 Fertility Rates

Fertility data is taken from Human Fertility Collection (2018). We use Age Specific Fertility Rates (ASFR) rather than Cumulative Period Fertility Rates (CPFR) to capture the number of births per thousand women for each age. We use the Age Reached During the Year (ARDY) definition of ages rather than Age in Completed Years (ACY) to ensure fertility rate data applies to only a single age group.

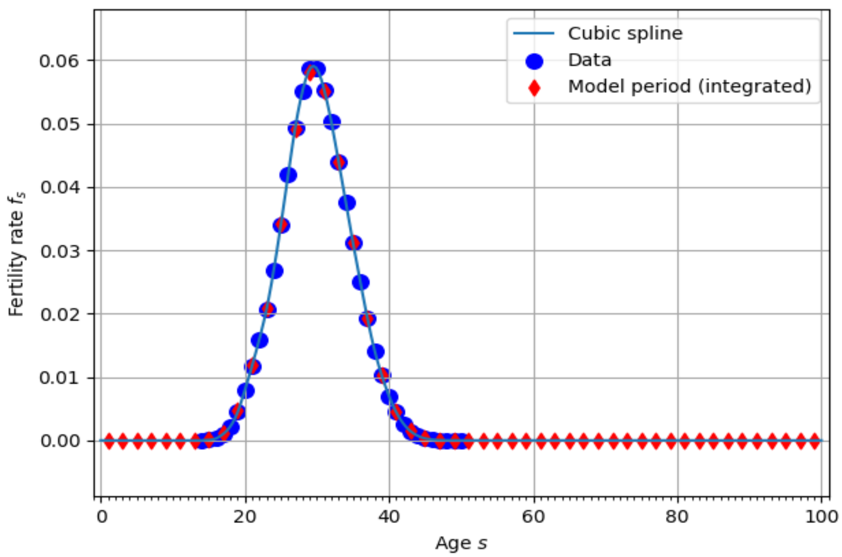
We assume a constant distribution of fertility rates over time. Our data come from 1990 to 2014 and provide the average fertility rates for each age from 14 to 50. We take an average over these time periods because we see high variation in Japanese fertility rate over the past few decades (not necessarily monotonically changing).[2] Therefore, using many years of data incorporates such variation in fertility rates over the years and consequently produces more

[2]Source: Holodny (2018)

realistic population dynamics.

Since our model does not include gender, we want fertility rates to represent the number of births per thousand population per age group. This requires us to divide fertility rates by two.[3] To provide flexibility to our model ages, we use cubic spline interpolation to fit fertility rates. This allows us to simulate our model using a variety of age bins through the use of population-weighted fertility rates. We interpolate population data for this weighting using data from Japanese Mortality Database (2018). The fit for one-year fertility rates can be seen in Figure 1.

Figure 1: Fertility Rates by Age (f_s) for $E + S = 100$



Source: Human Fertility Collection (2018)

3.1.3 Mortality Rates

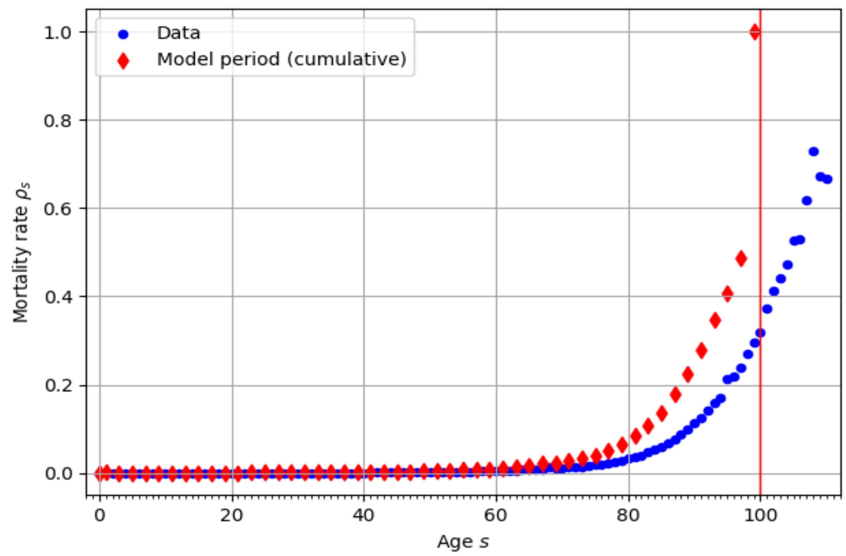
Mortality rates, ρ_s, represent the probability that a household that has reached age s will die before reaching age $s + 1$. As with fertility rates, we assume a constant distribution of mortality rates over time. Mortality data comes from Japanese Mortality Database (2018).

In order to match the mortality data, we do not use the most recent data from 2016 but instead use 2014 mortality rates. This includes mortality rates for each age from 0 to 110+. Our model assumes a mortality rate of 1.0 for age 100, so we drop data for older ages.

[3]We assume the population has an equal distribution of each gender.

We again use cubic spline interpolation to provide flexibility to our model ages. We take geometric means in order to properly adjust age bins as mortality rates are compounding. The fit for one-year mortality rates can be seen in Figure 2.

Figure 2: Mortality Rates by Age (ρ_s) for $E + S = 100$



Source: Japanese Mortality Database (2018)

3.1.4 Immigration Rates

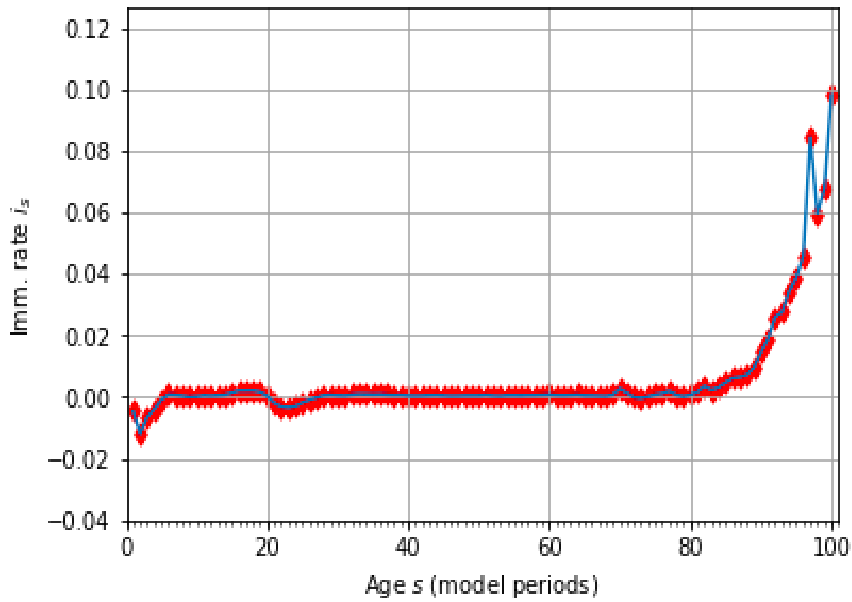
We calculate immigration rates as the residual between projected population and actual population. This is done by taking the population evolution equations described in 3.1.1 and solving for i. This leaves us with two equations:

$$i_1 = \frac{\omega_{1,t+1} - (1 - \rho_0) \sum_{s=1}^{E+S} f_s \omega_{s,t}}{\omega_{1,t}} \quad \forall t \tag{18}$$

$$i_{s+1} = \frac{\omega_{s+1,t+1} - (1 - \rho_s)\omega_{s,t}}{\omega_{s+1,t}} \quad \forall t \quad \text{and} \quad 1 \leq s \leq E + S - 1 \tag{19}$$

We use population data from 2011 to 2014 taken from Japanese Mortality Database (2018), the fertility rates from Section 3.1.2, and the mortality rates from Section 3.1.3 to calculate three immigration rates. We use the average over these three rates in our model. Our estimated immigration rates by age can be seen in Figure 3.

Figure 3: Immigration Rates by Age (i_s) for $E + S = 100$



Source: Japanese Mortality Database (2018)

3.1.5 Population Steady State and Transition Path

DeBacker and Evans (2018) provides a detailed description of the solution method for finding the steady state population distribution. They show that population will reach steady state population distribution with a unique eigenvector, $(1 + \bar{g}_n)$, which gives the steady-state population growth rate. We compare the theoretical steady-state population distribution to a steady-state distribution achieved through $t = 120$ iterations of the model in Figure 4. We show immigration rates such that Japan will realize steady state by period $t = 120$ in Figure 5, and stationary population distributions at periods along the transition path in Figure 6.

Figure 4: Theoretical Steady-State Population Distribution vs. Population Distribution at $t = 120$

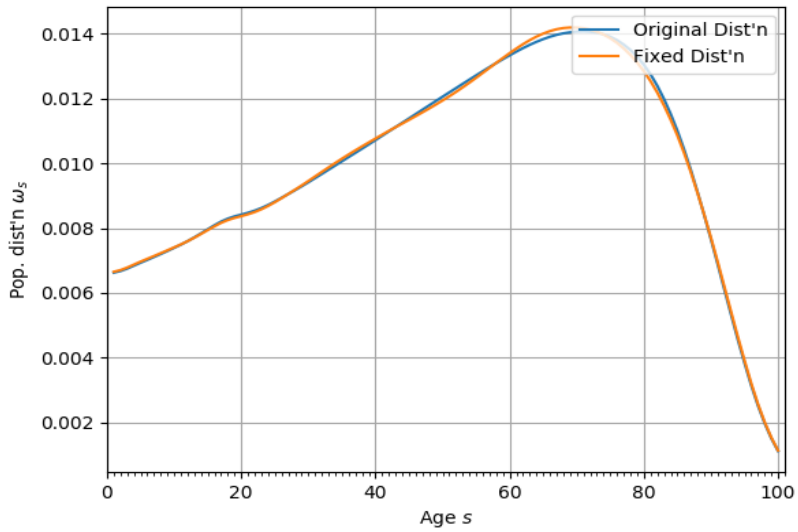


Figure 5: Original Immigration Rates vs Adjusted Immigration Rates to Make Fixed Steady-State Population Distribution

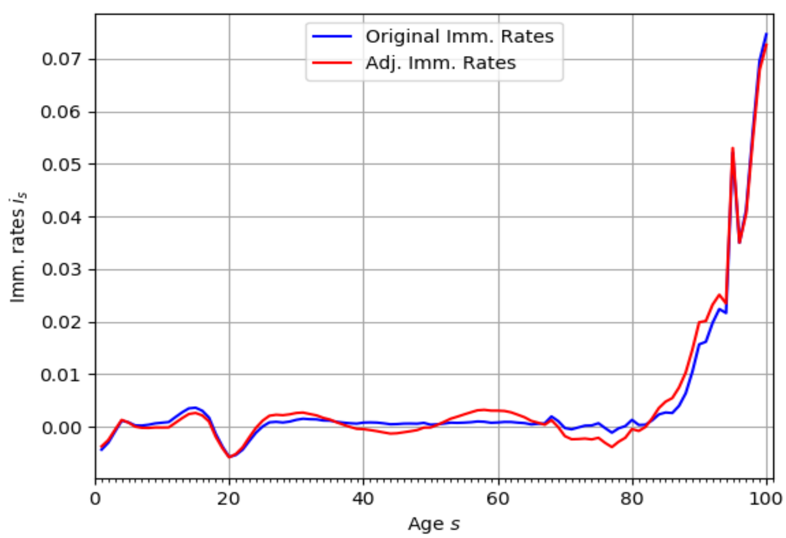
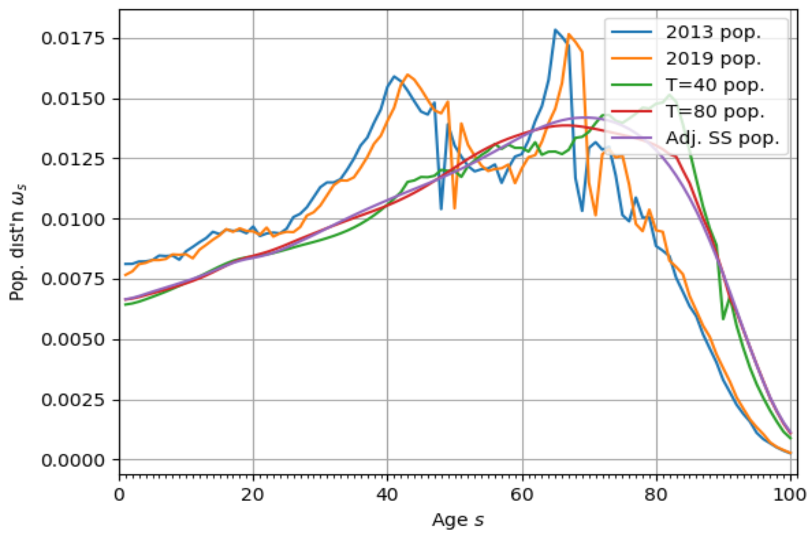


Figure 6: Stationary Population Distribution at Periods Along Transition Path



3.2 Elliptical Disutility of Labor Supply

We apply the method created in DeBacker and Evans (2018) of using the upper-right quadrant of ellipse in order to approximate the Constant Frisch Elasticity (CFE) disutility of labor. This functional form provides Inada conditions at both the upper bound ($\lim_{n\to 0}$) = 0) and lower bound ($\lim_{n\to\tilde{l}} = -\infty$). We represent the elliptical disutility of labor functional form as the following[4]:

$$g_{elp}(n_{s,t}) = -b\left[1 - \left(\frac{n_{s,t}}{\tilde{l}}\right)^v\right]^{\frac{1}{v}}$$

instead of Constant Frisch Elasticity (CFE) disutility of labor functional form:

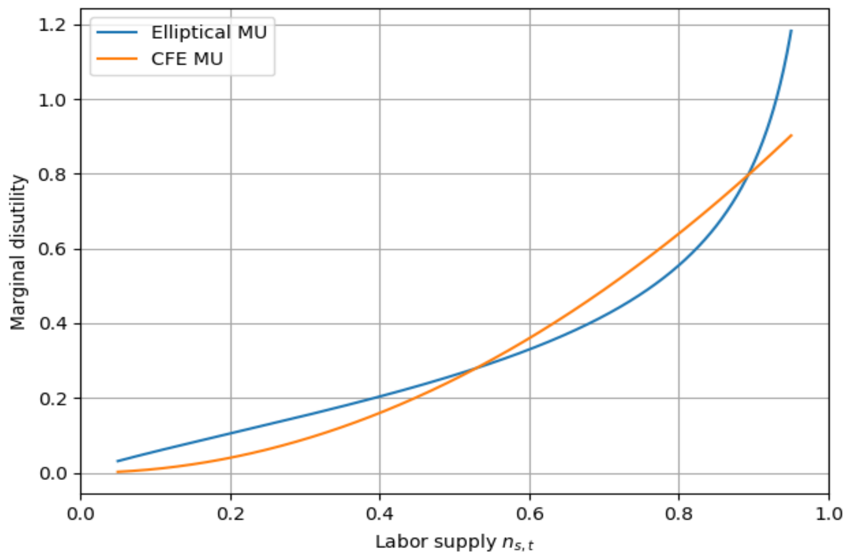
$$g_{cfe}(n_{s,t}) = \frac{(n_{s,t})^{1+\frac{1}{\theta}}}{1+\theta}$$

We use a Frisch elasticity of $\theta = 0.5$ as specified in Muto et al. (2016). The time endowment each period is $\tilde{l} = 0.73$ given the average sleep time in Japan.[5] To calibrate our elliptical functional form, we minimized sum of squared errors between the marginal utility of labor from the g_{elp} specification and the marginal utility of labor from the g_{cfe} specification. Our

[4] Please refer to DeBacker and Evans (2018) for a detailed description of the functional form derivation.
[5] The average sleep time in Japan is 6.5 hours. Source: Roll (2018)

calibrated parameters can be seen in Table 2.

Figure 7: Comparison of CFE Marginal Disutility of Leisure $\theta = 0.5$ to Fitted Elliptical Utility



3.3 Calibrating χ_s^n

We used a generalized method of moments calibration to choose χ_s^n such that steady-state labor model moments match recent-period labor data moments. Following DeBacker and Evans (2018), we define our labor moments as percent of total time endowment spent working by age. We only calibrate the χ_s^n using labor moments in the age range of 20 - 65. This is because Japan's retirement age is 65 and thus labor supply data moments decrease dramatically after 65. Such a dramatic shift cannot be explained by a change in preferences, and thus cannot be properly calibrated by χ_s^n. To calibrate χ_s^n for ages 65 and above, we simply extended our Chebyshev polynomial estimates using a linear approximation. The labor moments are in age bins of [20-24, 25-29, 30-34, 35-40, 40-45, 45-50, 50-55, 55-60].

$$\text{labor supply model moments}: m(\tilde{x}|\theta) = \frac{\bar{n_s}}{\tilde{l}} \quad \forall E \leq s \leq 65 \tag{20}$$

$$\text{labor supply data moments}: m(x) = \frac{\text{average hours worked}}{\text{total hours available}} \quad \forall E \leq s \leq 65 \tag{21}$$

To calculate the average hours worked by age for our data moments, we used the following data sources: Basic Survey on Wage Structure (2017) to find monthly average scheduled hours worked; and Labour Force Survey (2017) to find labor force participation rates, and average employment rates. These values are listed in Table 1. We extend the labor force participation rates and employment rates to fit the age bins of monthly average schedule hours worked.[6]

Given the data, we calculate average hours worked as:

$$\begin{aligned} \text{average hours worked} = \ & \text{labor force participation rate} \\ & \times \text{employment rate} \\ & \times \text{monthly hours worked} \end{aligned}$$

We then adjusted these values to be percent of total time endowment by using the average sleeping hours in Japan,[7] making $\tilde{l} = 7.3$. We set these labor data moments as corresponding to the midpoints of each age bins.

We used GMM estimation to calibrate χ^n_s by minimizing the distance between model moments and data moments, where we used the identity matrix ($W = I_4$) as the weighting matrix.

$$\begin{aligned} \hat{\theta}_{GMM} = \theta : \quad & \min_{\theta}\ e(\tilde{x}, x|\theta)^T\, W\, e(\tilde{x}, x|\theta) \\ \text{where } e(\tilde{x}, x|\theta) = & \frac{m(\tilde{x}|\theta) - m(x)}{m(x)} \end{aligned} \tag{22}$$

We fit a Chebyshev polynomial of degree 4 to calibrate χ^n_s for ages 20 - 65:

$$\begin{aligned} p(x) = c_0 + c_1 \cdot T_1(x) + c_2 \cdot T_2(x) + c_3 \cdot T_3(x) + c_4 \cdot T_4(x) \\ \text{where } T_n(x) = cos(ncos^{-1}(x)) \end{aligned} \tag{23}$$

[6] Extended average labor force participation rates of [0.69, 0.849, 0.849, 0.847, 0.847, 0.859, 0.859, 0.709, 0.709] correspond to age bins [20-24, 25-29, 30-34, 35-40, 40-45, 45-50, 50-55, 55-60]. Extended employment rates of [0.937, 0.954, 0.954, 0.966, 0.966, 0.97, 0.97, 0.968, 0.968] correspond to age bins [20-24, 25-29, 30-34, 35-40, 40-45, 45-50, 50-55, 55-60]. We extend rather than interpolate because there are so few data points and the data are already averages over these age bins.

[7] The average sleep time in Japan is 6.5 hours. Source: Roll (2018)

Our calibrated coefficients can be seen in Table 2. We use linear approximation to calibrate disutility of labor χ_s^n for ages 65 - 100 as extension of Chebyshev polynomial.

$$\chi_n = [p(65) - p(64)] \cdot (age - 65) + p(65) \tag{24}$$

Results of our calibration of χ_s^n using the Chebyshev polynomial and linear approximation are in Figure 8. Our labor moments can be seen in Figure 9.

We found that this calibration method is computationally inefficient. This is because the shape of the Chebyshev polynomial is very sensitive to small changes in the coefficient values. This makes it computationally difficult as it is calibrating all of the labor moments at the same time. Therefore, we came up with an alternative fixed point calibration method which we plan to include in future versions of the model. Please refer to Appendix A-6.

Figure 8: χ_s^n by Age for $E + S = 100$

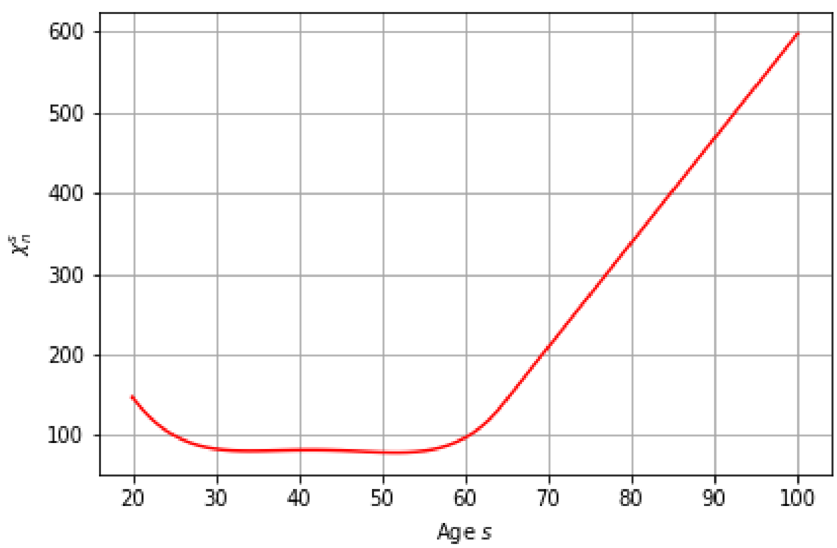
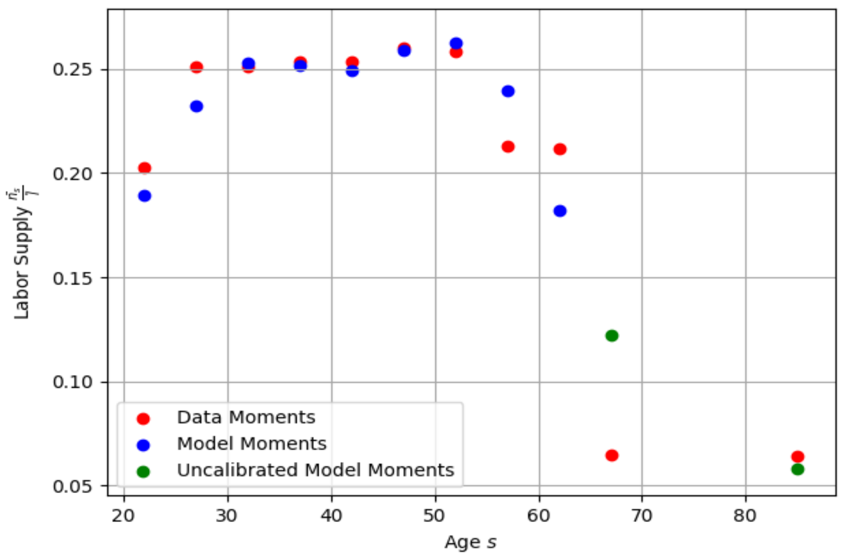


Figure 9: Labor Moments by Age ($\frac{\bar{n_s}}{\tilde{l}}$) for $E + S = 100$



3.4 Tax rates

Following DeBacker and Evans (2018), we define effective tax rate ($\tau_{s,t}^{ETR}$) as the total tax liability divided by total income (I).

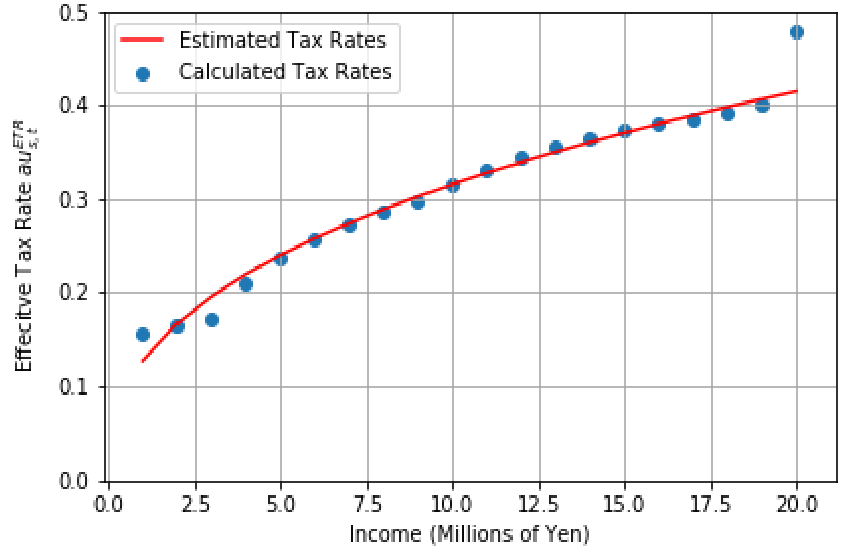
$$\tau^{ETR}(I) = \frac{T_{s,t}^{I}}{I} \tag{25}$$

We fit the Gouveia and Strauss (1994) tax function to the effective tax rates by income provided by Tax Guide Book (2018). The effective tax rate includes the labor income tax, capital income tax, and individual inhabitant tax.

$$T^{ETR,GS} = \phi_0 - \phi_0 \cdot (\phi_1 \cdot I^{\phi_2} + 1)^{(-1/\phi_2)} \tag{26}$$

We used GMM estimation to calibrate effective tax rates by minimizing the distance between model moments and data moments. This is the same approach as calibrating χ_s^n in Section 3.3. The calibrated coefficients are shown in Table 2.

Figure 10: Effective Tax Rate by Income Using GS (Millions of yen)

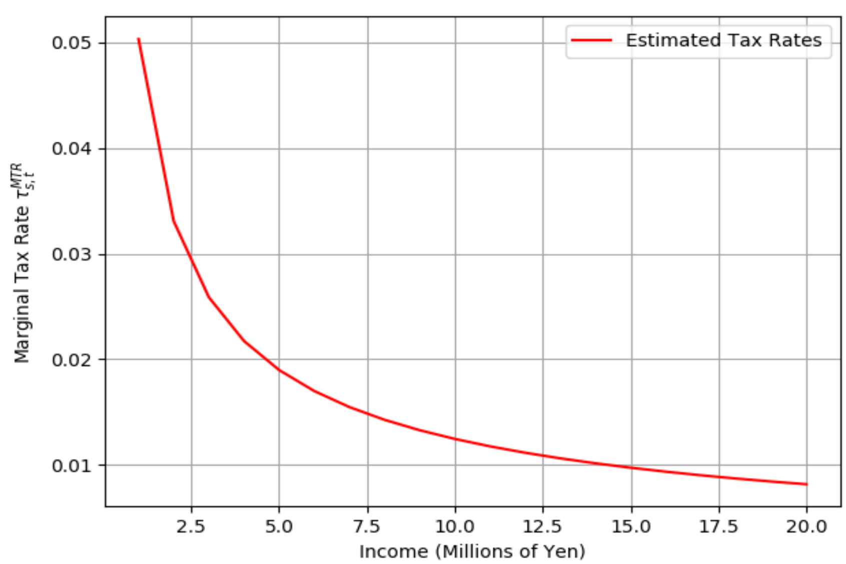


We analytically derive the marginal tax rate by taking the partial derivative of the Gouveia-Strauss calibration with respect to income:

$$\tau^{mtr} = \phi_0 \phi_1 I^{\phi_2 - 1} (\phi_1 I^{\phi_2} + 1)^{-\frac{1+\phi_2}{\phi_2}} \tag{27}$$

Our marginal tax rate estimates for different income levels are shown in Figure 11.

Figure 11: Analytical Marginal Tax Rate by Income (Millions of yen)



4 Steady State Results

Our steady state results can be seen in Table 3. This simulation includes our calibrated values for government spending, taxes, χ^n_s, and elliptical disutility of labor. Because this simulation does not include Time Path Iteration (TPI), we have no comparison for many of these variables and cannot interpret them. From the results we can interpret, of note is that government spending must become negative to meet the market-clearing conditions. This negative government spending implies that tax revenues are not enough to finance government spending in Japan. Therefore, government spending, which is around $\frac{3}{8}$ of GDP, could be interpreted as a capital influx from abroad to fund government spending. However, this explanation is not valid in our model as we simulate a closed economy. A possible solution to this impossible-to-interpret conclusion is to include government bonds, as in Muto et al. (2016). They argue that there is a tendency in Japan to hold government bonds over private sector bonds, which is why there is such a large spread on interest rates between the two. Including this in their model allows government debt payments to decrease significantly, which leads to positive government spending in the steady state. Including government bonds is a future extension to the model we intend to pursue. From our results, it appears that if the demographic transition continues, the current state of the financial system is unsustainable.

5 Conclusion

We investigate the aggregate macroeconomic effect of population aging in Japan using the overlapping generations model provided by DeBacker and Evans (2018). Our steady state results imply that with the estimated demographic transition given the current fertility rate and mortality rate, the Japanese financial system will not be sustainable in the long-run. Government spending must be negative at the steady-state to meet the market-clearing conditions. Negative government spending implies that tax revenues are not enough to finance government spending in our model. Therefore, as an extension, we plan to include government bonds as specified in Muto et al. (2016). They explain that including a preference for government bonds will lower interest rates on government borrowing. This makes it easier for the government to finance its debt.

For further extensions, we intend to run Time Path Iteration (TPI) in order to see the transition from the current level of the economy to the steady-state. This will allow us to interpret our steady-state results relative to current levels. We also plan to have the productivity level of households $e_{j,s}$ calibrated to the Japanese productivity level. Our calibrations for individual productivity levels can be seen in Appendix A-5. Moreover, we plan to re-calibrate the χ_s^n by using the fixed point calibration method explained in Appendix A-6. Additionally, Muto et al. (2016) include the Japanese healthcare industry which they argue causes underestimates of the costs of aging if it is not included. We hope to include this in future versions of the model.

Some potential drawbacks of the paper include using ability levels calibrated to US data. We have a dataset of Japanese ability levels but it only gives one datapoint per age, while the US calibration gives seven ability levels per age. We do not expect updating the calibration to change our results much, as US ability levels and Japanese ability levels should generally be close. We also use US calibrations for the corporate income tax. We believe that this may be underestimating the tax burden placed on businesses which may be contributing to the negative government spending in steady-state.

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APPENDIX

A-1 Derivation

To calculate government transfer and discretionary spending as percent of GDP, we first take $G = 97,454.7$ billion yen[8] and $Y = 546,489$ billion yen[9]:

$$\Leftrightarrow \frac{G}{Y} = 0.1783287495 = 17.8\%$$

From World Bank (2018b), 68.977% of government spending is transfer payments. This is 12.3% of GDP.

From Japanese Public Finance Fact Sheet (2017), 24.1% of government spending is debt service. Subtracting this from the spending remaining after transfers puts discretionary spending at 6.923% of total government spending. This is 1.23% of GDP.

A-2 Data Tables

Table 1: Labor Data

Age Bin	LFPR	Employment Rate	Age Bin	Hours Worked (Monthly)	Labor Efficiency
20-24	0.69	0.937	20-24	167	0.646
25-34	0.849	0.954	25-29	165	0.834
			30-34	165	0.999
35-44	0.847	0.966	35-39	165	1.107
			40-44	166	1.165
45-54	0.859	0.97	45-49	165	1.218
			50-54	165	1.233
55-64	0.709	0.968	55-59	164	1.127
			60-64	-	0.820
65+	-	-	65+	-	0.727

Sources: Labour Force Survey (2017); Basic Survey on Wage Structure (2017); Braun et al. (2009).

[8] Source: Japanese Public Finance Fact Sheet (2017)

[9] Source: World Bank (2018a)

A-3 Calibration Parameter Estimates

Table 2: Calibration Parameter Estimates

Type	Parameter	Estimate
Government Spending	α_{tr}	0.1230058215
	α_g	0.01234569933
Elliptical Disutility of Labor	b	0.4082468175957912
	v	1.858718515674512
Chebyshev Function	c_0	1.10807470e+03
	c_1	-1.05805189e+02
	c_2	1.92411660e+00
	c_3	-1.53364020e-02
	c_4	4.51819445e-05
Gouveia-Strauss Calibrated Coefficients	ϕ_0	5.66885717e+01
	ϕ_1	8.89727694e-04
	ϕ_2	3.96781859e-01
Ability Level Extrapolation Parameters	a	18.11499367
	b	0.13083097
	c	0.3428676
	d	27.13388349

A-4 Steady State Results

Table 3: Steady State Results

Symbols	Description	Value
Y_{ss}	Steady state output	0.4048560175726948
G_{ss}	Steady state government spending	-0.1495370364999672
C_{ss}	Steady state aggregate consumption	0.4464433295861147
K_{ss}	Steady state aggregate capital demand	1.5914364315115423
I_{ss}	Steady state aggregate investment	0.10794972448636408
L_{ss}	Steady state aggregate labor demand	0.19372751175324024
Debt Service		0.709
D/Y	Debt to GDP ratio	2.000000000000592
T/Y	Tax revenue to GDP ratio	0.12347844280006419
G/Y	Government spending to GDP ratio	-0.36935856207970713
Maximum error in labor FOC: 2.611244553918368e-13		
Maximum error in savings FOC: 2.611244553918368e-13		
Run time: 1412.5289669036865 seconds		

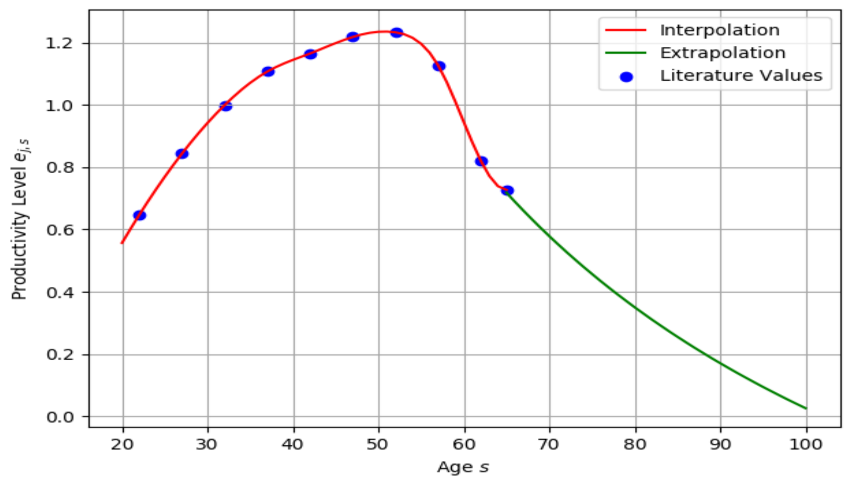
A-5 Individual Productivity Level $e_{j.s}$

We use labor efficiency profiles constructed by Braun et al. (2009). These values can be seen in Table 1. We interpolate to generate labor efficiency profiles for ages 20 - 65. For ages 65 - 80, we estimate by fitting an *arctan* function of the following form, which is a slight adjustment of the form used by DeBacker and Evans (2018):

$$y = -a \cdot \arctan(b \cdot x + c) + d \tag{A.5.1}$$

where x is age and a, b, c, d are parameters. Our parameter estimates are in Table 2.

Figure 12: Ability Level $e_{j,s}$

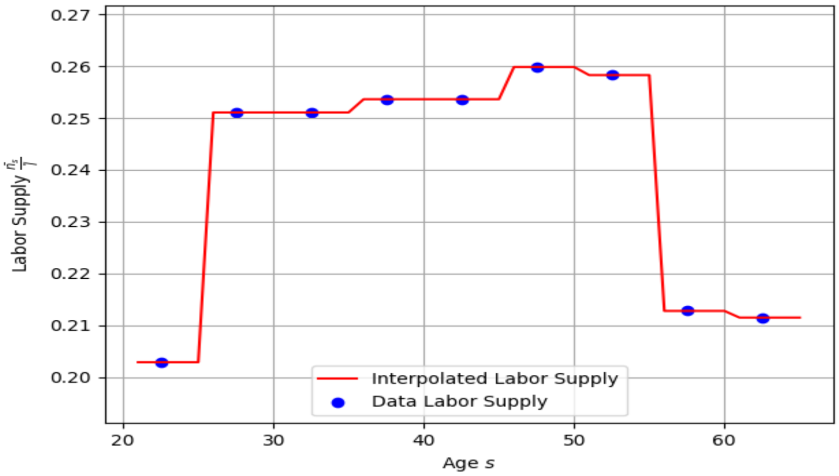


A-6 New χ^n_s Calibration Method

This section describes our calibration method for χ^n_s. In order to more efficiently solve for χ^n_s, we propose a fixed point solution method.

Our labor data moments are calculated the same way as Section 3.3. However, instead of taking the midpoint of each age bin, we treat each age group as having labor data moments equal to the average over the entire age group $E+1 \leq s \leq E+S$. Denote the data moments for age s as $\bar{n}_{s,data}$. The interpolated labor data moments are in Figure 13.

Figure 13: Interpolated Labor Data Moments $\bar{n}_{s,data}$



We outline the steps of the solution method below.

i. Guess initial values $\chi^n_{s,guess}$.

ii. Given $\chi^n_{s,guess}$, we find the steady state labor supply $n_{j,s}$ for all ability types j and $E+1 \leq s \leq E+S$. We take an average over all ability types to get $\bar{n}_{s,model}$ for all $E+1 \leq s \leq E+S$. This is because we do not have labor data moments for different ability types and only the average over the entire population in Japan.

iii. Define $error_s = |\bar{n}_{s,model} - \bar{n}_{s,data}|$. If $error_s < \epsilon$ for some $\epsilon > 0$ and for all s, use $\chi^n_{s,guess}$. Otherwise, continue to step (iv).

iv. If $\bar{n}_{s,model} > \bar{n}_{s,data}$, set

$$\bar{n}^{above}_{s,model} = min(\bar{n}_{s,model}, \bar{n}^{above}_{s,model})$$

and

$$\chi^n_{s,above} = max(\chi^n_{s,guess}, \chi^n_{s,above})$$

If $\bar{n}_{s,model} < \bar{n}_{s,data}$, set

$$\bar{n}^{below}_{s,model} = max(\bar{n}_{s,model}, \bar{n}^{below}_{s,model})$$

and

$$\chi^n_{s,below} = min(\chi^n_{s,guess}, \chi^n_{s,below})$$

We now have upper and lower bounds for our estimates.

v. Given upper and lower bounds on χ^n_s, generate a new vector of $\chi^n_{s,guess}$ as a convex combination of $\chi^n_{s,below}$ and $\chi^n_{s,above}$ based on the relative distance of the labor moments from the data moments. Return to step (iii).

We applied the fixed point calibration method with $error_s \leq 0.03$. The results are in Figure 14. It took 240 minutes to get these results. We can see that the labor model moments are within 0.03 of the labor data moments.

Figure 14: Interpolated Labor Data Moments $\bar{n}_{s,data}$