HOUSING MARKETS AND OPTIMAL PRICING STRATEGY

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Abstract

We construct a structural model for single-family house selling which relies on sale price, time on the market, and latent listing price in a simplified economy. We perform an optimization with our model to determine optimal listing price given housing covariates. This has implications on how house sellers can maximize their profit when moving houses by gaming in an imperfect information system. We also use Maximum Likelihood techniques to recover model parameters from simulated real estate data.

1 Introduction

We present a simplified model of imperfect-information house selling which depends on time on the market, housing characteristics, and listing price. There is a growing literature which has established an empirical relationship between listing price and time on the market (TOM) [1]. We suppose that our given data includes the final selling price and time taken on the market to sell for each house, along with covariates. We further restrict to a relatively small number of house types to simulate the sell-off of a finished group of housing developments. In the model, sellers list their house on the market at a price based on a function of house characteristics, but with some noise (due to heterogeneous preferences, sentimental value, regret, etc.) [2]. As the house stays on the market, final selling price decreases over time due to the associated costs of real estate management, advertising, open houses, etc., but can also increase in the short run through competitive auctions between buyers.

Duration on the market is modeled similarly to standard continuous arrival times, with distribution of arrival times being dependent on the perceived gap in value of the house. In general, if a house is listed far below its perceived worth, it should sell off very quickly. Conversely, if a house is listed far above its perceived worth, it should be expected to spend more time on the market before being sold (as buyers bargain with the seller to reduce costs). Further, we should expect some covariates to determine the influence of the value gap on this distribution. There arises an opportunity for gaming the market here because the seller might list the house for higher than it's worth, then when the house is sold at closing price it is lower than listing price but still higher than actual house value. This implies that sellers could solve an optimization problem to determine the listing price which maximizes their profit; we explicate and solve this problem below.

It is reasonable to assume that data of this type could be accessed for application of the model. County auditors keep public records of sale price and closing dates for individual houses. These data are aggregated and organized by companies like Title plants, which "keep track of land titles and all related documents," like sales data [3]. An extension of this paper would use real data to calibrate our model, but such data is not currently available to us.

2 Structural Model

$\mathbb{X}_i$, the covariates of housing, are created as follows:

$$\mathbb{X}_i = \{\#beds, \#baths, size = \{\text{small}, \text{med}, \text{large}\}\}$$

$$size = P(\text{“small”}, \text{“med”}, \text{“large”}; p_1, p_2, p_3)$$

$$baths = beds = \begin{cases} rand(1,2) & size == \text{small} \\ rand(2,3) & size == \text{med} \\ rand(3,4) & size == \text{large} \end{cases}$$

This creates 12 distinct types/models of houses, in a similar manner to how a group of housing developments might choose the characteristics of the houses they create. This process allows for randomness in the data while also imposing realistic restrictions on house characteristics. In our simulation we assume $p_1 = p_2 = p_3 = (1/3)$ but we could change these probabilities to easily reflect a different distribution of house sizes in the economy.

We begin by assuming that listing price is Normally distributed according to some function of house characteristics:

$$P_i^L \sim \mathcal{N}(f(\mathbb{X}_i\beta), \sigma_L).$$

The function f allows us to alter where the distribution of P_i^L is centered, so for example we could add a quantity to $\mathbb{X}_i\beta$ to reflect an initial higher offer by the seller; this could reflect a game-theoretic approach for house selling. We assume that f is the identity function in our simulation.

The **Value Gap** in the house is defined as

$$V_i(P_i^L) = P_i^L - \mathbb{X}_i\beta,$$

i.e. the difference between the listing price and the actual value of the house. In our simulation, $\mathbb{E}[V_i] = 0$, but for real seller behavior we would expect to observe $\mathbb{E}[V_i] > 0$.

Duration on the market (TOM) follows an Exponential distribution as a function of the Value Gap:

$$\lambda_i(V_i) = \left(\frac{\alpha}{e^{V_i}} - c\right)^{1/(1+\gamma)} > 0$$

$$D_i \sim Exponential(\lambda_i),$$

where α, γ are economic coefficients of interest, and c may be some associated cost. Each house's duration on the market is a random draw from an exponential distribution which depends on the value gap as well as economic conditions. This distribution accounts for the ideas posed above:

$$P_i^L \gg \mathbb{X}_i\beta \implies \lambda \approx 0 \implies Pr(D_i \leq \zeta) \text{ small} \quad \text{for } \zeta \in \mathbb{R}$$

$$P_i^L \ll \mathbb{X}_i\beta \implies \lambda \gg 0 \implies Pr(D_i \leq \zeta) \text{ large} \quad \text{for } \zeta \in \mathbb{R}$$

Finally, selling price is a function of both listing price (P_i^L) and market Duration:

$$P_i = P_i^L - \eta(D_i)^{1/(1+\epsilon)},$$

again with associated economic coefficients of interest. This implies that final selling price is decreasing in duration with associated costs: if the house were to sell immediately, it would sell at listing price, while if the house takes a long time to sell, its final price will be significantly lower than listing price. But this also allows for a gaming opportunity in P_i^L. In the next section, we analyze this opportunity with an optimization approach.

3 Optimal Selling

From our model parameter estimates, we can now find the optimal listing price given house characteristics $\mathbb{X}_i$. We maximize the following expected selling price P_i:

$$\begin{aligned}\max_{P_i^L}\left\{\mathbb{E}[P_i^*|P_i^L,\mathbb{X}_i]\right\} &= \max_{P_i^L}\left\{\mathbb{E}\left(P_i^L - \eta(D_i)^{1/(1+\epsilon)} \mid P_i^L,\mathbb{X}_i\right)\right\} \\ &= \max_{P_i^L}\left\{P_i^L - \eta\mathbb{E}\left((D_i)^{1/(1+\epsilon)} \mid P_i^L,\mathbb{X}_i\right)\right\}\end{aligned}$$

In the simpler case where $\epsilon = 0$ and $c = 0$, then we can get a closed form solution for P_i^{L*}:

$$P_i^L = \log(\alpha) + (1+\gamma)\log(1+\gamma) - (1+\gamma)\log(\eta) + \mathbb{X}_i\beta$$

See Appendix A.1 for the derivation.

We also present an algorithm for computation of the general case:

1. Calculate

$$P_i^L - \eta\mathbb{E}\left[D^{1/(1+\epsilon)} \mid P_i^L, X_i\right] = P_i^L - \eta\int_{\mathbb{R}^+} D_i^{1/(1+\epsilon)}\cdot\lambda_i e^{-\lambda_i D_i}\, dD_i.$$

2. Then maximize:

$$\begin{aligned} 0 &= \frac{\partial}{\partial P_i^L}P_i^L - \eta\int_{\mathbb{R}^+} D_i^{1/(1+\epsilon)}\cdot\lambda_i e^{-\lambda_i D_i}\, dD_i \\ \frac{1}{\eta} &= \frac{\partial}{\partial P_i^L}\int_{\mathbb{R}^+} D_i^{1/(1+\epsilon)}\cdot\lambda_i e^{-\lambda_i D_i}\, dD_i.\end{aligned}$$

3. Compute the expression above in terms of P_i^L.

We use this algorithm to create plots over potential values of P_i^L in Figure 1 and Figure 2.

4 Results/Conclusion

Through our paper we recover the parameters of our model by using Maximum Likelihood estimation strategy. Our derivation of the ML estimator can be seen in Appendix A.1. Because our model contains nine parameters as well as an integral over $\mathbb{R}$, the ML estimator is computationally taxing. We were able to run the MLE using a sampled dataset with 20 observations. The results can be seen in Table 1.

For MLE, the identification condition is that $\forall \theta \neq \theta', L(\theta|\mathbf{X}) \neq L(\theta'|\mathbf{X})$. When setting the initial guess near the true parameters, we recover estimates near the true parameters as shown in Table 1. However, when setting the initial guess randomly, we do not necessarily recover the true parameters. We believe that the log-likelihood function has many local maxima, and thus we may not converge to the global maximum unless we guess parameter values close to the true values. We also believe this could be caused by small sample bias, as we are simulating only 20 data points. Therefore, in practice, we can take many different initial guesses and take the estimates that give the highest log-likelihood value.

Table 1: **Parameter Estimates**

Parameter	DGP	Estimates
β	[2, 1.75, 3]	[2.92, 1.63, 2.49]
σ	1	0.57
α	1	1.00
c	0	5.57e-06
γ	1	1.00
ϵ	1	1.20
η	10	10.07
Observation Number: 20		
Log-likelihood: -34.48		
Run time: 22 min 24 secs		

For MLE, we can construct relevant standard errors using the Fisher information matrix. In theory, we can solve this as the following:

$$var(\theta) = [I(\theta)]^{-1}$$
$$I(\theta) = -E[H(\theta)]$$
$$H(\theta) = \frac{\partial^2 \ln \mathcal{L}(\theta)}{\partial\theta\partial\theta'}$$

Therefore, we have the variance-covariance matrix as the following:

$$var(\theta^{MLE}) = \left(-E\left[\frac{\partial^2 \ln \mathcal{L}(\theta)}{\partial\theta\partial\theta'}\right]\right)^{-1}$$

In our paper, we computed the standard errors numerically. They can be seen in Table 2. The standard errors are large relative to the coefficients because of the small sample size.

Table 2: **Standard Errors**

Parameter	Std. err.
β	[3.30 2.49 0.75]
σ	0.31
α	1.0
c	1.0
γ	1.0
ϵ	0.80
η	1.96

We present graphs of our general model in the Appendix. Keeping parameters around the truth, Figure 1 plots selling price for a range of listing prices, for all possible house types. The graphs all exhibit the predicted behavior based on intuition: as listing price increases, selling price initially increases (due to gaming), attains a maximum, then quickly decreases (due to costs and buyer attitude). In Figure 2, we perform comparative statics on the parameters $(\alpha, \epsilon, \eta, \gamma)$ for a representative household type. To analyze the effects of the parameters, we restrict to the simplified-case optimal listing price (P_i^L) when $\epsilon = 0$ and $c = 0$, so we can use the closed-form solution for P_i^L:

$$P_i^L = \log(\alpha) + (1+\gamma)\log(1+\gamma) - (1+\gamma)\log(\eta) + \mathbb{X}_i\beta \tag{*}$$

- From Figure 1, we observe that better housing characteristic $\mathbb{X}_i$ increases the optimal listing price. This is expected from our analytical expression for P_i^L from (*).
- From Figure 2, we observe that increasing α strictly increases the optimal listing price. This is expected from our analytical expression for P_i^L from (*).
- From Figure 2, we observe that increasing η strictly decreases the optimal listing price. This is expected from our analytical expression for P_i^L from (*).
- From Figure 2, the effect of increasing γ on optimal listing price depends on the value of γ relative to the other parameters. We derive the relationship between P_i^L and γ in Appendix A.5. The results are the following:

$$P_i^L \begin{Bmatrix} \textit{decreasing} \\ \textit{constant} \\ \textit{increasing} \end{Bmatrix} \text{ in } \gamma \text{ if } \begin{Bmatrix} \gamma < \log^{-1}(\log(\eta) - 1) - 1 \\ \gamma = \log^{-1}(\log(\eta) - 1) - 1 \\ \gamma > \log^{-1}(\log(\eta) - 1) - 1 \end{Bmatrix}$$

- From Figure 2, the effect of increasing ϵ on optimal listing price is unclear. We cannot use the simplified equation (*) because it assumes $\epsilon = 0$.

Our results reflect the game-theoretic and auction-theoretic motivations of the model. A house seller can increase their final selling price up to a certain point by increasing their listing price above the value of the house. This comes from the idea that buyers have high demand for the house and, in a relatively crowded market, are willing to take a suboptimal offer to beat out their competitors. The model also reflects market friction, since no house is sold immediately on the market; there are factors which slow down the process of selling the house. To further extend the game-theoretic implications of the model, we could let the above model represent the first stage in a multi-stage game of market selling which alternates between seller and buyer strategies each round. This would of course become much more complicated but would better reflect the back-and-forth nature of market exchange behavior.

Since the MLE strategy is computationally expensive, we hope to use expection-maximization (EM) estimation strategy going forward. Our derivation of the EM algorithm can be seen in Appendix A.3. One reason we are having numerical difficulties is because in the MLE log-likelihood expression, we have log of an integral. However, by using Jenson's inequality through EM algorithm, we can make the expected log-likelihood have the log inside the integral. We also have code to implement our algorithm, but we could not complete running the code given the time frame.

A Appendix

A.1 Optimal Selling

Derivation for optimal list price in the simple case ($\epsilon = c = 0$):

$$\begin{aligned}
\mathbb{E}\left(P_i^L - \eta D_i^{1/(1+\epsilon)} \mid P_i^L, \mathbb{X}_i\right) &= P_i^L - \eta \mathbb{E}[D_i \mid P_i^L, \mathbb{X}_i] \\
&= P_i^L - \eta \cdot \frac{1}{\lambda_i} \\
&= P_i^L - \eta \cdot \left(\frac{e^{V_i} - \overbrace{c}^{=0}}{\alpha}\right)^{1/(1+\gamma)} \\
&= P_i^L - \eta \cdot \left(\frac{e^{P_i^L - \mathbb{X}_i\beta}}{\alpha}\right)^{1/(1+\gamma)}
\end{aligned}$$

$$\begin{aligned}
\mathbb{E}[P_i^* | P_i^L] &= \max_{P_i^L} P_i^L - \eta \cdot \left(\frac{e^{P_i^L - \mathbb{X}_i\beta}}{\alpha}\right)^{1/(1+\gamma)} \\
\iff 1 - \frac{\eta}{\alpha(1+\gamma)} \left(\frac{e^{P_i^L - \mathbb{X}_i\beta}}{\alpha}\right)^{-\gamma/(1+\gamma)} &\cdot e^{P_i^L - \mathbb{X}_i\beta} = 0 \\
\frac{\alpha(1+\gamma)}{\eta} = \left(\frac{e^{P_i^L - \mathbb{X}_i\beta}}{\alpha}\right)^{-\gamma/(1+\gamma)} &\cdot e^{P_i^L - \mathbb{X}_i\beta} \\
\frac{\alpha(1+\gamma)}{\eta} &= \alpha^{\gamma/(1+\gamma)} e^{(P_i^L - \mathbb{X}_i\beta)(1-\gamma/(1+\gamma))} \\
\alpha^{1/(1+\gamma)} \frac{1+\gamma}{\eta} &= e^{(P_i^L - \mathbb{X}_i\beta)(1/(1+\gamma))} \\
\log\left(\alpha^{1/(1+\gamma)} \frac{1+\gamma}{\eta}\right) &= (P_i^L - \mathbb{X}_i\beta)(1/(1+\gamma)) \\
P_i^L &= (1+\gamma)\log\left(\alpha^{1/(1+\gamma)} \frac{1+\gamma}{\eta}\right) + \mathbb{X}_i\beta \\
P_i^L &= \log(\alpha) + (1+\gamma)\log(1+\gamma) - (1+\gamma)\log(\eta) + \mathbb{X}_i\beta
\end{aligned}$$

A.2 Maximum Likelihood Derivation

For each individual, we have the following likelihood function:

$$\begin{aligned}\mathcal{L}_i(P_i, D_i, \mathbb{X}_i|\theta) &= f(P_i|D_i, \mathbb{X}_i, \theta) \cdot f(D_i, \mathbb{X}_i|\theta) \\ &= f(P_i|D_i, \mathbb{X}_i, \theta) \cdot f(D_i|\mathbb{X}_i, \theta) \cdot f(\mathbb{X}_i|\theta) \\ &= f(P_i|D_i, \mathbb{X}_i, \theta) \cdot f(\mathbb{X}_i|\theta) \cdot \int_{-\infty}^{\infty} f(D_i, P_i^L|\mathbb{X}_i, \theta) dP_i^L \\ &= f(P_i|D_i, \mathbb{X}_i, \theta) \cdot f(\mathbb{X}_i|\theta) \cdot \int_{-\infty}^{\infty} f(D_i|P_i^L, \mathbb{X}_i, \theta) \cdot f(P_i^L|\mathbb{X}_i, \theta) dP_i^L\end{aligned}$$

Here, assume that $\{P_i, D_i, \mathbb{X}_i\}$ are iid over observations. So, for N individuals, the likelihood function becomes the following:

$$\mathcal{L}(\{P_i, \mathbb{X}_i, D_i\}_i^N|\theta) = \Pi_{i=1}^N f(P_i|D_i, \mathbb{X}_i, \theta) \cdot f(\mathbb{X}_i|\theta) \cdot \int_{-\infty}^{\infty} f(D_i|P_i^L, \mathbb{X}_i, \theta) \cdot f(P_i^L|\mathbb{X}_i, \theta) dP_i^L$$

Here, we have the following functional form:

$$P_i = P_i^L - \eta D_i$$

We know that the distribution for P_i^L has the following distribution:

$$P_i^L \sim \mathcal{N}(f(\mathbb{X}_i\beta), \sigma_L)$$

Therefore, we have the following distribution for P_i when D_i is known:

$$P_i|D_i \sim \mathcal{N}(f(\mathbb{X}_i\beta) - \eta D_i^{\frac{1}{1+\epsilon}}, \sigma_L)$$

So, the likelihood becomes:

$$\begin{aligned}\mathcal{L}(P_i, \mathbb{X}_i, D_i|\theta) = \Pi_{i=1}^N \frac{1}{\sigma_L\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{P_i - f(\mathbb{X}_i\beta) + \eta D_i^{1/(1+\epsilon)}}{\sigma_L}\right)^2\right) \\ \cdot \int_{-\infty}^{\infty} \left(\frac{\alpha}{\mathbb{X}_i\beta - P_i^L} - c\right)^{1/(1+\gamma)} \cdot \exp\left(-\left(\frac{\alpha}{\mathbb{X}_i\beta - P_i^L} - c\right)^{1/(1+\gamma)} \cdot D_i\right) \\ \cdot \frac{1}{\sigma_L\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{P_i^L - f(\mathbb{X}_i\beta)}{\sigma_L}\right)^2\right) dP_i^L\end{aligned}$$

$$
\begin{aligned}
\log \mathcal{L}(P_i, \mathbb{X}_i, D_i|\theta) = & \sum_{i=1}^{n} \log \left(\frac{1}{\sigma_L \sqrt{2\pi}} \exp \left(-\frac{1}{2} \left(\frac{P_i - f(\mathbb{X}_i \beta) + \eta D_i^{1/(1+\epsilon)}}{\sigma_L} \right)^2 \right) \right) \\
& + \log \left(\int_{-\infty}^{\infty} \left(\frac{\alpha}{\mathbb{X}_i \beta - P_i^L} - c \right)^{1/(1+\gamma)} \cdot \exp \left(- \left(\frac{\alpha}{\mathbb{X}_i \beta - P_i^L} - c \right)^{1/(1+\gamma)} \cdot D_i \right) \right. \\
& \left. \cdot \frac{1}{\sigma_L \sqrt{2\pi}} \exp \left(-\frac{1}{2} \left(\frac{P_i^L - f(\mathbb{X}_i \beta)}{\sigma_L} \right)^2 \right) dP_i^L \right) \\
= & \sum_{i=1}^{n} \log \left(\frac{1}{\sigma_L \sqrt{2\pi}} \exp \left(-\frac{1}{2} \left(\frac{P_i - f(\mathbb{X}_i \beta) + \eta D_i^{1/(1+\epsilon)}}{\sigma_L} \right)^2 \right) \right) \\
& + \log \left(\int_{-\infty}^{\infty} \lambda_i \exp \left(-\lambda_i \cdot D_i \right) \cdot \frac{1}{\sigma_L \sqrt{2\pi}} \exp \left(-\frac{1}{2} \left(\frac{V_i}{\sigma_L} \right)^2 \right) dP_i^L \right) \\
= & \sum_{i=1}^{n} -\log \Big(\sigma_L \sqrt{2\pi} \Big) - \frac{1}{2} \left(\frac{P_i - f(\mathbb{X}_i \beta) + \eta D_i^{1/(1+\epsilon)}}{\sigma_L} \right)^2 \\
& - \log \Big(\sigma_L \sqrt{2\pi} \Big) + \log \left(\int_{-\infty}^{\infty} \lambda_i \exp \left(-\lambda_i \cdot D_i \right) \exp \left(-\frac{1}{2} \left(\frac{V_i}{\sigma_L} \right)^2 \right) dP_i^L \right) \\
= & \sum_{i=1}^{n} -2 \log \Big(\sigma_L \sqrt{2\pi} \Big) - \frac{1}{2} \left(\frac{P_i - f(\mathbb{X}_i \beta) + \eta D_i^{1/(1+\epsilon)}}{\sigma_L} \right)^2 \\
& + \log \left(\int_{-\infty}^{\infty} \lambda_i \exp \left(-\lambda_i \cdot D_i \right) \exp \left(-\frac{1}{2} \left(\frac{V_i}{\sigma_L} \right)^2 \right) dP_i^L \right)
\end{aligned}
$$

A.3 Expectation Maximization Derivation

In the expectation step, we compute:

$$
\begin{aligned}
P(P_i^L|P_i, D_i, \mathbb{X}_i, \theta^\tau) &= \frac{P(P_i^L, P_i, D_i|\mathbb{X}_i, \theta^\tau)}{P(P_i, D_i|\mathbb{X}_i, \theta^\tau)} \\
&= \frac{P(P_i^L, P_i, D_i|\mathbb{X}_i, \theta^\tau)}{P(P_i|D_i, \mathbb{X}_i, \theta^\tau) \cdot P(D_i|\mathbb{X}_i, \theta^\tau)} \\
&= \frac{P(P_i|P_i^L, D_i, \mathbb{X}_i, \theta^\tau) \cdot P(P_i^L, D_i|\mathbb{X}_i, \theta^\tau)}{P(P_i|D_i, \mathbb{X}_i, \theta^\tau) \int_{-\infty}^{\infty} P(D_i, P_i^L|\mathbb{X}_i, \theta^\tau) dP_i^L} \\
&= \frac{P(P_i|P_i^L, D_i, \mathbb{X}_i, \theta^\tau) \cdot P(D_i|P_i^L, \mathbb{X}_i, \theta^\tau) P(P_i^L|\mathbb{X}_i, \theta^\tau)}{P(P_i|D_i, \mathbb{X}_i, \theta^\tau) \cdot \int_{-\infty}^{\infty} P(D_i|P_i^L, \mathbb{X}_i, \theta^\tau) \cdot P(P_i^L|\mathbb{X}_i, \theta^\tau) dP_i^L}
\end{aligned}
$$

In the maximization step, we maximize the following:

$$
\begin{aligned}
Q(\theta|\theta^\tau) &= \left(\sum_{i=1}^{N} E[\log[P(P_i, D_i, P_i^l|\mathbb{X}_i, \theta)]|P_i, D_i, \mathbb{X}_i, \theta^\tau] \right) \\
&= \left(\sum_{i=1}^{N} \int_{-\infty}^{\infty} P(P_i^l|P_i, D_i, \mathbb{X}_i, \theta^\tau) \log[P(P_i, D_i, P_i^l|\mathbb{X}_i, \theta)] dP_i^l \right) \\
&= \left(\sum_{i=1}^{N} \int_{-\infty}^{\infty} P(P_i^l|P_i, D_i, \mathbb{X}_i, \theta^\tau) \log[P(P_i, D_i|P_i^l, \mathbb{X}_i, \theta) P(P_i^l|\mathbb{X}_i, \theta)] dP_i^l \right) \\
&= \left(\sum_{i=1}^{N} \int_{-\infty}^{\infty} P(P_i^l|P_i, D_i, \mathbb{X}_i, \theta^\tau) \log[P(P_i|D_i, P_i^l, \mathbb{X}_i, \theta) P(D_i|P_i^l, \mathbb{X}_i, \theta) P(P_i^l|\mathbb{X}_i, \theta)] dP_i^L \right)
\end{aligned}
$$

A.4 Comparative Statics

Figure 1: Graphs of Listing Price vs Selling Price for all types of houses, by house size

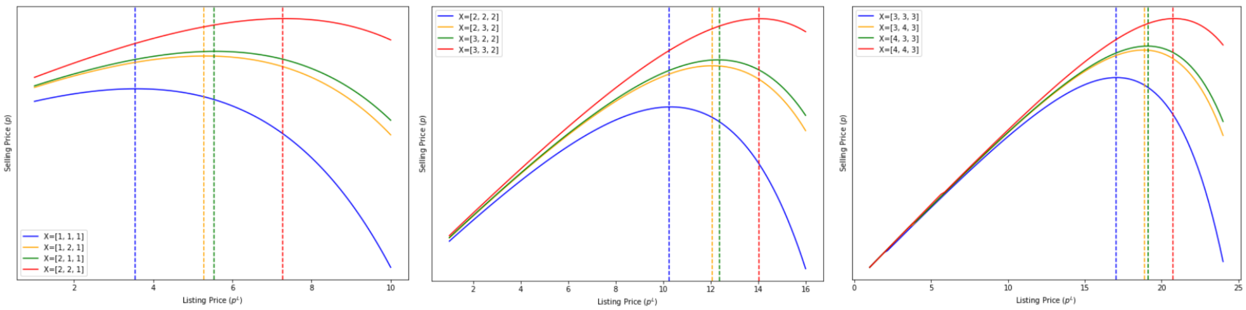
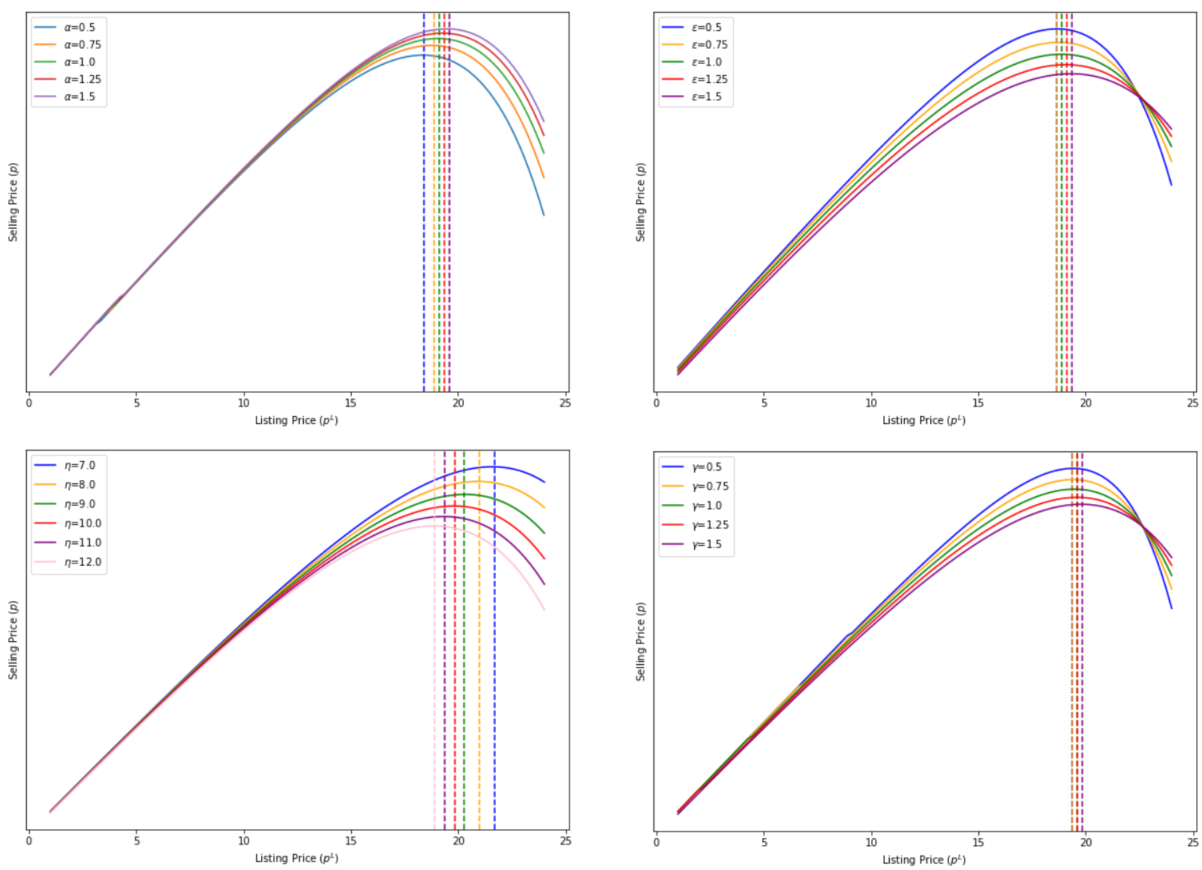


Figure 2: Graphs of Listing Price vs Selling Price for a representative household, varying certain parameters



A.5 Effect of γ on Optimal P_i^L

We have

$$\begin{aligned} P_i^L &= \log(\alpha) + (1+\gamma)\log(1+\gamma) - (1+\gamma)\log(\eta) + \mathbb{X}_i\beta \\ \left[\frac{\partial}{\partial\gamma}\right] : RHS &= \log(1+\gamma) + \frac{1+\gamma}{1+\gamma} - \log(\eta) \\ &= \log(1+\gamma) + 1 - \log(\eta) \end{aligned}$$

So we get:

$$RHS \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0 \text{ if } \begin{Bmatrix} \gamma < \log^{-1}(\log(\eta) - 1) - 1 \\ \gamma = \log^{-1}(\log(\eta) - 1) - 1 \\ \gamma > \log^{-1}(\log(\eta) - 1) - 1 \end{Bmatrix}$$

This leads us to:

$$P_i^L \begin{Bmatrix} decreasing \\ constant \\ increasing \end{Bmatrix} \text{ in } \gamma \text{ if } \begin{Bmatrix} \gamma < \log^{-1}(\log(\eta) - 1) - 1 \\ \gamma = \log^{-1}(\log(\eta) - 1) - 1 \\ \gamma > \log^{-1}(\log(\eta) - 1) - 1 \end{Bmatrix}$$

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